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The contribution of off-shell gluons to the longitudinal structure function F_L

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Abstract. We present the results for the structure function $F_{\rm L}$ for a gluon target having a non-zero transverse momentum square at order $\alpha_{\rm s}$. The results of a double convolution (with respect to the Bjorken variable x and the transverse momentum) of the perturbative part and the unintegrated gluon densities are compared with recent experimental data for $F_{\rm L}$ at low x values and with the predictions of other approaches.

1 Introduction

The basic information on the internal structure of nucleons is extracted from the process of deep inelastic (lepton–hadron) scattering (DIS). Its differential cross section has the form

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}y} = \frac{2\pi\alpha_{\rm em}^2}{xQ^4} \times \left[\left(1-y+y^2/2\right)F_2(x,Q^2) - \left(y^2/2\right)F_{\rm L}(x,Q^2)\right],$$

where $F_2(x,Q^2)$ and $F_{\rm L}(x,Q^2)$ are the transverse and longitudinal structure functions (SF), respectively, q^μ and p^μ are the photon and the hadron 4-momenta, and $x = Q^2/(2pq)$ with $Q^2 = -q^2 > 0$.

The longitudinal SF $F_{\rm L}(x,Q^2)$ is a very sensitive QCD characteristic, because it is equal to zero in the parton model with spin$-1/2$ partons. Unfortunately, essentially at small values of x, experimental extraction of $F_{\rm L}$ data requires rather a cumbersome procedure (see [1, 2], for example). Moreover, the perturbative QCD leads to some controversial results in the case of SF $F_{\rm L}$. The next-to-leading order (NLO) corrections to the longitudinal coefficient function, which are large and negative at small x [3, 4], need a resummation procedure[1] which leads to a coupling constant scale essentially higher than Q^2 [4, 6, 7][2].

Recently, there have become available important new data [13–18] on the longitudinal SF $F_{\rm L}$ which probed the small-x region down to $x \sim 10^{-2}$. Moreover, at small x the SF $F_{\rm L}$ can be related to the SF F_2 and the derivative $\mathrm{d}F_2/\mathrm{d}\ln(Q^2)$ (see [19–21]). In this way, most precise predictions based on data for F_2 and $\mathrm{d}F_2/\mathrm{d}\ln(Q^2)$ (see [15] and references therein) can be obtained for $F_{\rm L}$. These predictions can be considered as indirect "experimental data" for $F_{\rm L}$.

In this paper, for the analysis of the above data we use the so-called $k_{\rm T}$-factorization approach [22–24] based on the BFKL dynamics [12] (see also the recent review of [11] and the references therein). In the framework of the $k_{\rm T}$-factorization approach, the longitudinal SF $F_{\rm L}$ has first been studied in [25], where the small x asymptotics of $F_{\rm L}$ has been evaluated using the BFKL results for the Mellin transform of the unintegrated gluon distribution and the longitudinal Wilson coefficient functions have been calculated analytically for the full perturbative series at asymptotically small x values. Since we want to analyze $F_{\rm L}$ data in a broader range at small x, we use the parameterizations of the unintegrated gluon distribution function $\Phi_g(x,k_\perp^2)$ (see Sect. 3).

The unintegrated gluon distribution $\Phi_g(x,k_\perp^2)$ (f_g is the (integrated) gluon distribution in the proton multiplied by x and $k_\perp$ is the transverse part of the gluon 4-momentum k^μ), with

$$f_{\rm g}(x,Q^2) = \int^{Q^2} \mathrm{d}k_\perp^2 \Phi_g(x,k_\perp^2) \qquad (1)$$

(hereafter $k^2 = -k_\perp^2$), is the basic dynamic quantity in the $k_{\rm T}$-factorization approach[3]. It satisfies the BFKL equation [12].

[1] Without a resummation the NLO approximation of the SF $F_{\rm L}$ can be negative at low x and quite low Q^2 values (see [4, 5])

[2] Note that at low x a similar property has also been observed in the approaches of [8–10] (see the recent review in [11] and discussions therein) which is based on Balitsky–Fadin–Kuraev–Lipatov (BFKL) dynamics [12], where the leading $\ln(1/x)$ contributions are summed

[3] In our previous analysis [26] we have shown that the property $k^2 = -k_\perp^2$ leads to the equality of the Bjorken x value in the standard renormalization-group approach and in the Sudakov one

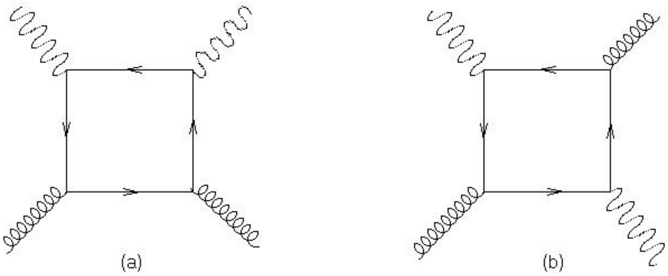

Fig. 1a,b. The diagrams contributing to $T_{\mu\nu}$ for a gluon target. They should be multiplied by a factor of 2 because of the opposite direction of the fermion loop. The diagram in **a** should be also doubled because of crossing symmetry

Notice that the integral is divergent at the lower limit (at least, for some parameterizations of $\Phi_g(x,k_\perp^2)$) and so it leads to the necessity to consider the difference $f_g(x, Q^2) - f_g(x,Q_0^2)$ with some non-zero Q_0^2 (see the discussions in [26]), i.e.

$$f_g(x,Q^2) = f_g(x,Q_0^2) + \int_{Q_0^2}^{Q^2} \mathrm{d}k_\perp^2 \Phi_g(x,k_\perp^2). \qquad (2)$$

Then in the k_T-factorization the SFs $F_{2,\mathrm{L}}(x,Q^2)$ are driven at small x primarily by gluons and are related in the following way to the unintegrated distribution $\Phi_g(x,k_\perp^2)$:

$$F_{2,\mathrm{L}}(x,Q^2) = \int_x^1 \frac{\mathrm{d}z}{z} \int \mathrm{d}k_\perp^2 \sum_{i=u,d,s,c} e_i^2 \cdot \hat{C}^g_{2,\mathrm{L}}(x/z,Q^2,m_i^2,k_\perp^2)\Phi_g(z,k_\perp^2), \qquad (3)$$

where e_i^2 are the charges squared of the active quarks.

The functions $\hat{C}^g_{2,\mathrm{L}}(x,Q^2,m_i^2,k_\perp^2)$ can be regarded as the SF of the off-shell gluons with virtuality $k_\perp^2$ (hereafter we call these *hard structure functions*[4]). They are described by the sum of the quark box (and crossed box) diagram contribution to the photon–gluon interaction (see Fig. 1).

The purpose of this paper is to give predictions for the longitudinal SF $F_\mathrm{L}(x,Q^2)$ based on the calculations of the hard SFs $\hat{C}^g_{2,\mathrm{L}}(x,Q^2,m^2,k_\perp^2)$, given in our previous study [26], and several parameterizations of the unintegrated gluon distributions (see [11] and references therein).

It is instructive to note that the diagrams shown in Fig. 1 are similar to those of the photon–photon scattering process. The corresponding QED contributions have been calculated many years ago in [27] (see also the beautiful review in [28]). Our results have been calculated independently and they are in full agreement with [27]. Moreover, our results are in agreement with the corresponding integral representations for $\hat{C}^g_{2,\mathrm{L}}$, given in [22, 25] and numerically with the results of [29]. However, we hope that our formulae, which are given in a simpler form, could be useful for others. This simpler form for the hard SFs $\hat{C}^g_{2,\mathrm{L}}$ comes from using the relation between the results based on nonsense, transverse and longitudinal gluon polarizations (see (13) below) observed in [26] for gauge-invariant sets of diagrams.

[4] By analogy with similar relations between cross-sections and hard the cross-sections

The structure of this paper is as follows: in Sect. 2 we present the basic formulae of our approach. Section 3 contains the relations between the SFs F_L and F_2 and the derivative $\mathrm{d}F_2/\mathrm{d}\ln Q^2$, obtained in [19–21] in the framework of the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) approach [30] (i.e., in the collinear approximation: $k_\perp^2 = 0$). In Sect. 4 we give the predictions for the structure function F_L for three cases of unintegrated gluon distributions.

2 Basic formulae

To begin with, we shortly review the results of [26] needed below in our investigations.

The hadron part of the DIS spin-average lepton–hadron cross section can be represented in the form[5]

$$F_{\mu\nu} = e_{\mu\nu}(q)\ F_\mathrm{L}(x,Q^2) + d_{\mu\nu}(q,p)F_2(x,Q^2), \qquad (4)$$

where

$$e_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

and

$$d_{\mu\nu}(q,p) = -\left[g_{\mu\nu} + 2x\frac{(p_\mu q_\nu + p_\nu q_\mu)}{q^2} + p_\mu p_\nu \frac{4x^2}{q^2}\right].$$

2.1 Feynman-gauge gluon polarization

As it has been shown in [26], it is very convenient to consider, as a first approximation, gluons having polarization tensor (hereafter the indices α and β are connected with gluons and μ and ν are connected with photons)[6]:

$$\hat{P}^{\alpha\beta} = -g^{\alpha\beta}. \qquad (5)$$

The tensor corresponds to the standard choice of the polarization matrix in the framework of the collinear approximation. In a sense the case of polarization is equal to the standard DIS suggestions for the parton properties, except for their off-shell property. The polarization (5) gives the main contribution to the polarization tensor we are interested in (see below):

$$\hat{P}^{\alpha\beta}_\mathrm{BFKL} = \frac{k_\perp^\alpha k_\perp^\beta}{k_\perp^2}, \qquad (6)$$

[5] Hereafter, we consider only the one-photon exchange approximation

[6] In principle, we can use here more general cases of the polarization tensor (for example, one is based on the Landau or unitary gauge). The difference between them and (5) is $\sim k^\alpha$ and/or $\sim k^\beta$ and, hence, it leads to zero contributions because the Feynman diagrams on Fig. 1 are gauge invariant

which comes from the high energy (or k_T)-factorization prescription [22–24][7].

Contracting the photon projectors (connected with the photon indices of the diagrams on Fig. 1)

$$\hat{P}^{(1)}_{\mu\nu} = -\frac{1}{2}g_{\mu\nu} \quad \text{and} \quad \hat{P}^{(2)}_{\mu\nu} = 4z^2\frac{k_\mu k_\nu}{Q^2}$$

(here $z = Q^2/(2kq)$ is the corresponding Bjorken variable at the parton level), with the parton tensor $F^p_{\mu\nu}$

$$F^p_{\mu\nu} = e_{\mu\nu}(q)\ F^p_{\rm L}(z,Q^2) + d_{\mu\nu}(q,k)F^p_2(z,Q^2), \qquad (7)$$

we obtain at the parton level (i.e. for off-shell gluons having momentum k_μ) when $\hat{C}^g_{2,\rm L}(z) \sim F^p_{2,\rm L}(z,Q^2)$[8]

$$\tilde{\beta}^2 \cdot \hat{C}^{g,(1)}_2(x) = \mathcal{K}\cdot\left[f^{(1)} + \frac{3}{2\tilde{\beta}^2}\cdot f^{(2)}\right], \qquad (8)$$

$$\begin{aligned}\tilde{\beta}^2 \cdot \hat{C}^{g,(1)}_{\rm L}(x) &= \mathcal{K}\cdot\left[4bx^2 f^{(1)} + \frac{(1+2bx^2)}{\tilde{\beta}^2}\cdot f^{(2)}\right]\\ &= \mathcal{K}\cdot f^{(2)} + 4bx^2\tilde{\beta}^2\cdot\hat{C}^{g,(1)}_2, \end{aligned}\qquad (9)$$

where the normalization factor is $\mathcal{K} = a_{\rm s}(Q^2)\cdot x$,

$$\hat{P}^{(i)}_{\mu\nu}F_{\mu\nu} = \mathcal{K}\cdot f^{(i)}, \quad i=1,2,$$

and $a_{\rm s}(Q^2) = \alpha_{\rm s}(Q^2)/(4\pi)$, $\tilde{\beta}^2 = 1-4bx^2$, $b = -k^2/Q^2 \equiv k^2_\perp/Q^2 > 0$, $a = m^2/Q^2$.

Applying the projectors $\hat{P}^{(i)}_{\mu\nu}$ to the Feynman diagrams displayed in Fig. 1, we obtain the following results:

$$\begin{aligned}f^{(1)} &= -2\beta\Big[1\\ &- (1-2x(1+b-2a)\cdot[1-x(1+b+2a)])\cdot f_1\\ &+ (2a-b)(1-2a)x^2\cdot f_2\Big],\end{aligned}\qquad (10)$$

$$\begin{aligned}f^{(2)} &= 8x\cdot\beta\Big[(1-(1+b)x)\\ &- 2x(bx(1-(1+b)x)(1+b-2a) + a\tilde{\beta}^2)\cdot f_1\\ &+ bx^2(1-(1+b)x)(2a-b)\cdot f_2\Big],\end{aligned}\qquad (11)$$

where

$$\beta^2 = 1 - \frac{4ax}{(1-(1+b)x)} \qquad (12)$$

and

$$f_1 = \frac{1}{\tilde{\beta}\beta}\cdot\ln\frac{1+\beta\tilde{\beta}}{1-\beta\tilde{\beta}}, \quad f_2 = -\frac{4}{1-\beta^2\tilde{\beta}^2}.$$

[7] We would like to note that the BFKL-like polarization tensor (6) is a particular case of the so-called nonsense polarization of particles in the t-channel. The nonsense polarization makes the main contributions to the cross sections in the s-channel as $s\to\infty$ (see, for example, [31] and references therein). The limit $s\to\infty$ corresponds to small values of the Bjorken variable x, which is just the range of our study

[8] The hard SFs $\hat{C}^g_{2,\rm L}$ do not depend on the type of target, so we can replace $z\to x$ below

2.2 BFKL-like gluon polarization

Now we take into account the BFKL-like gluon polarization (6). As we have shown in [26], the projector $\hat{P}^{\alpha\beta}_{\rm BFKL}$ can be represented as

$$\hat{P}^{\alpha\beta}_{\rm BFKL} = -\frac{1}{2}\frac{1}{\tilde{\beta}^4}\left[\tilde{\beta}^2 g^{\alpha\beta} - 12bx^2\frac{q^\alpha q^\beta}{Q^2}\right]. \qquad (13)$$

In the previous subsection we have already presented the contributions to the hard SF using the first term in the brackets of the r.h.s. of (13). Repeating the above calculations with the projector $\sim q^\alpha q^\beta$, we obtain the total contributions to the hard SF which can be represented as the following shift of the results in (8)–(11):

$$\begin{aligned}&\hat{C}^{g,(1)}_2(x) \to \hat{C}^g_2(x), \quad \hat{C}^{g,(1)}_{\rm L}(x) \to \hat{C}^g_{\rm L}(x);\\ &f^{(1)} \to f^{(1)}_{\rm BFKL} = \frac{1}{\tilde{\beta}^4}\left[\tilde{\beta}^2 f^{(1)} - 3bx^2\tilde{f}^{(1)}\right]\\ &f^{(2)} \to f^{(2)}_{\rm BFKL} = \frac{1}{\tilde{\beta}^4}\left[\tilde{\beta}^2 f^{(2)} - 3bx^2\tilde{f}^{(2)}\right],\end{aligned}\qquad (14)$$

where

$$\begin{aligned}\tilde{f}^{(1)} &= -\beta\Bigg[\frac{1-x(1+b)}{x}\\ &- 2(x(1-x(1+b))(1+b-2a) + a\tilde{\beta}^2)\cdot f_1\\ &- x(1-x(1+b))(1-2a)\cdot f_2\Bigg],\end{aligned}\qquad (15)$$

$$\begin{aligned}\tilde{f}^{(2)} &= 4\cdot\beta(1\\ &- (1+b)x)^2[2-(1+2bx^2)\cdot f_1 - bx^2\cdot f_2].\end{aligned}\qquad (16)$$

Notice that the general formulae are needed only to evaluate the charm contribution to the structure functions F_2 and $F_{\rm L}$, i.e. F^c_2 and $F^c_{\rm L}$. To evaluate the corresponding light quark contributions, i.e. $F^l_{\rm L}$, we can use the $m^2=0$ limit of the above formulae.

2.3 The case $m^2 = 0$

When $m^2 = 0$, the hard SFs $\hat{C}^g_k(x)$ are defined by $f^{(1)}$, $f^{(2)}$, $\tilde{f}^{(1)}$ and $\tilde{f}^{(2)}$ (as in (8), (9) and (14)) which can be represented as

$$\begin{aligned}f^{(1)} &= -2\Big[2\\ &- (1-2x(1+b)+2x^2(1+b)^2)\cdot L(\tilde{\beta})\Big],\end{aligned}\qquad (17)$$

$$f^{(2)} = 8x(1+b)(1-(1+b)x)\Big[1-2bx^2\cdot L(\tilde{\beta})\Big], \qquad (18)$$

$$\begin{aligned}\tilde{f}^{(1)} &= -\frac{(1+b)(1-x(1+b))}{bx}[1-2bx^2\cdot L(\tilde{\beta})]\\ &= -\frac{1}{8bx^2}f^{(2)},\end{aligned}\qquad (19)$$

$$\tilde{f}^{(2)} = 4(1-x(1+b))^2[3-(1+2bx^2)\cdot L(\tilde{\beta})], \qquad (20)$$

and, thus, (see (14))

$$\begin{aligned}
\tilde{\beta}^4 \cdot f^{(1)}_{\rm BFKL} &= (-2)\,(1 - x(1+b)) \\
&\times \left[2\left(1 - 2x(1+b) + \frac{x^2(1-b)^2}{1-x(1+b)}\right) \right. \\
&- \bigl(1 - x(1+b) - 4x^3 b(1+b) \\
&+ \left. \frac{x^2(1-b)^2}{1-x(1+b)}\right) \cdot L(\tilde{\beta}) \Bigg], \qquad (21)
\end{aligned}$$

$$\begin{aligned}
\tilde{\beta}^4 \cdot f^{(2)}_{\rm BFKL} &= 8x(1 - x(1+b)) \\
&\times \Bigl[1 + b - 18bx(1 - x(1+b)) \\
&+ 2bx\,(3 - 4x(1+b) \\
&+ 6bx^2(1 - x(1+b))) \cdot L(\tilde{\beta}) \Bigr], \qquad (22)
\end{aligned}$$

where

$$L(\tilde{\beta}) = \frac{1}{\tilde{\beta}} \cdot \ln \frac{1+\tilde{\beta}}{1-\tilde{\beta}}.$$

3 Relations between $F_{\rm L}$, F_2 and the derivative of F_2 in the case of the collinear approximation

More additional information about the SF $F_{\rm L}$ can be obtained in the collinear approximation (i.e. when $k_\perp^2 = 0$) in the following way.

In the framework of perturbative QCD, there is a possibility of connecting $F_{\rm L}$ with F_2 and with its derivative $\mathrm{d}F_2/\mathrm{d}\ln Q^2$ due to the fact that at small x the DIS structure functions depend only on two *independent* functions: the gluon distribution and the singlet quark one (the non-singlet quark density is negligible at small x), which in turn can be expressed in terms of the measurable SF F_2 and its derivative $\mathrm{d}F_2/\mathrm{d}\ln Q^2$.

In this way, by analogy with the case of the gluon distribution function (see [32, 33] and references therein), the behavior of $F_{\rm L}(x,Q^2)$ has been studied in [19–21], using the HERA data of [34, 35] and the method of [36][9] consisting of the replacement of the Mellin convolution by ordinary products. Thus, the small x behavior of the SF $F_{\rm L}(x,Q^2)$ can be extracted directly from the measured values of $F_2(x,Q^2)$ and its derivative without a cumbersome procedure (see [1, 2]). These extracted values of $F_{\rm L}$ may be well considered as *new small x "experimental data" of* $F_{\rm L}$. The relations can be violated by non-perturbative corrections like higher-twist ones [38, 39], which can be large exactly in the case of the SF $F_{\rm L}$ [25, 40].

Since the $k_{\rm T}$-factorization approach is one of the popular perturbative approaches used at small x, it is very useful to compare its predictions with the results of [19–21] based on the relations between the SFs $F_{\rm L}(x,Q^2)$, $F_2(x,Q^2)$ and $\mathrm{d}F_2(x,Q^2)/\mathrm{d}\ln Q^2$. It is the main purpose of this study.

The $k_{\rm T}$-factorization approach is closely related to the Regge-like behavior of the parton distributions. So we restrict our investigations to the SF and parton distributions in the following form (hereafter $a = q, g$):

$$f_a(x,Q^2) \sim F_2(x,Q^2) \sim x^{-\delta(Q^2)}. \qquad (23)$$

Note that really the slopes of the sea quark and gluon distributions δ_q and δ_g, respectively, and the slope δ_{F2} of the SF F_2 are somewhat different. The slopes have the familiar property $\delta_q < \delta_{F2} < \delta_g$ (see [41–46] and references therein). We will neglect, however, this difference and use in our investigations the experimental values of $\delta(Q^2) \equiv \delta_{F2}(Q^2)$ extracted by the H1 Collaboration[10] (see [41] and references therein). We note that the Q^2-dependence is in very good agreement with perturbative QCD at $Q^2 \geq 2\,\mathrm{GeV}^2$ [48]. Moreover, the values of the slope $\delta(Q^2)$ are in agreement with recent phenomenological studies (see, for example, [8]) incorporating the next-to-leading corrections [49] (see also [50]) in the framework of the BFKL approach.

Thus, assuming the *Regge-like behavior* (23) for the gluon distribution and $F_2(x,Q^2)$ at $x^{-\delta} \gg 1$ and using the NLO approximation for the collinear coefficient functions and the anomalous dimensions of the Wilson operators, the following results for $\mathrm{F_L}(x,Q^2)$ have been obtained in [20]:

$$\begin{aligned}
F_{\rm L}(x,Q^2) &= -2\frac{B_{\rm L}^{g,1+\delta}(1 + a_{\rm s}(Q^2)\overline{R}_{\rm L}^{g,1+\delta})}{\gamma_{qg}^{(0),1+\delta} + \overline{\gamma}_{qg}^{(1),1+\delta} a_{\rm s}(Q^2)} \xi^\delta \\
&\times \left[\frac{\mathrm{d}F_2(x\xi,Q^2)}{\mathrm{d}\ln Q^2} \right. \\
&+ \left. \frac{a_{\rm s}(Q^2)}{2}\left(\frac{B_{\rm L}^{q,1+\delta}}{B_{\rm L}^{g,1+\delta}}\gamma_{qg}^{(0),1+\delta} - \gamma_{qq}^{(0),1+\delta}\right) F_2(x\xi,Q^2) \right] \\
&+ O(a_{\rm s}^2, x^{2-\delta}, \alpha x^{1-\delta}), \qquad (24)
\end{aligned}$$

where

$$\begin{aligned}
\overline{\gamma}_{qg}^{(1),\eta} &= \gamma_{qg}^{(1),\eta} + B_2^{q,\eta}\gamma_{qg}^{(0),\eta} + B_2^{g,\eta}(2\beta_0 + \gamma_{gg}^{(0),\eta} - \gamma_{qq}^{(0),\eta}), \\
\overline{R}_{\rm L}^{g,\eta} &= R_{\rm L}^{g,\eta} - B_2^{g,\eta}\frac{B_{\rm L}^{q,\eta}}{B_{\rm L}^{g,\eta}}.
\end{aligned}$$

Here $\overline{\gamma}_{qa}^{(1),\eta}$ and $\overline{R}_{\rm L}^{a,\eta}$ $(a = q, g)$ are the combinations[11] of the "anomalous dimensions" of the Wilson operators

[9] This method is based on previous investigations [1, 37]

[10] Now the preliminary ZEUS data for the slope $\mathrm{d}\ln F_2/\mathrm{d}\ln(1/x)$ are available as some points on Figs. 8 and 9 in [47]. Moreover, the new preliminary H1 points have been presented at the Workshop DIS2002 [17]. Both new points show quite similar properties to compare with the H1 data [41]. Unfortunately, the tables of the ZEUS data and the new H1 data are unavailable yet, so the points cannot be used here

[11] Because we consider here $F_2(x,Q^2)$ but not the singlet quark distribution in the corresponding DGLAP equations [30]

$\gamma_{qa}^{\eta} = a_{\rm s}\gamma_{qa}^{(0),\eta} + a_{\rm s}^2\gamma_{qa}^{(1),\eta} + O(a_{\rm s}^3)$ and the "Wilson coefficients" $a_{\rm s}B_{\rm L}^{a,\eta}(1+a_{\rm s}R_{\rm L}^{a,\eta}) + O(a_{\rm s}^3)$ and $a_{\rm s}B_2^{a,\eta} + O(a_{\rm s}^2)$ with the "moment" argument η (i.e., the combinations of the functions which can be obtained by analytical continuation of the corresponding anomalous dimensions and coefficient functions from integer values n of their argument to non-integer ones η).

Note that, in principle, any term like $\sim 1/(n+m)$ $(m = 0,1,2,\dots)$ which adds to the corresponding combinations of the anomalous dimensions and coefficient functions: $\overline{\gamma}_{qa}^{(1),n}$ and $\overline{R}_{\rm L}^{a,n}$, should contribute to (24) in the following form (after replacement of Mellin convolutions by usual products in the DGLAP equations (see [36])):

$$\frac{1}{1+\delta+m}\left(1+\frac{\Gamma(2+\delta+m)\Gamma(1+\nu)}{\Gamma(2+\delta+m+1+\nu)}\cdot x^{1+\delta+m}\right), \quad (25)$$

where the value of ν comes [51–53] from the asymptotics of the parton distributions $f_a(x)$ as $x \to 1$: $f_a \sim (1-x)^{\nu_a}$, and[12] $\nu \approx 4$ from the quark counting rules [54]. The additional term $\sim x^{1+\delta+m}$ in (25) is important only for the $m = -1$ case (i.e. for the singular parts $\sim 1/(n-1)$ of the corresponding anomalous dimensions and coefficient functions) and quite small values of δ (i.e., for $x^\delta \sim$ Const).

Thus, except for the case when $m = -1$ and $x^\delta \sim$ Const, we can replace (25) by its first term $1/(1+\delta+m)$, i.e., our variables $\overline{\gamma}_{qg}^{(1),\eta}$ and $\overline{R}_{\rm L}^{g,\eta}$ are just the combinations of the corresponding anomalous dimensions and coefficient functions at $n = 1+\delta$. When $m = -1$ and $x^\delta \sim$ Const in the small x range considered here, we should replace the term $1/(n-1)$, if any existed in the variables $\overline{\gamma}_{qa}^{(1),n}$ and $\overline{R}_{\rm L}^{a,n}$ $(a = q,g)$, by the following term:

$$\frac{1}{\tilde{\delta}} = \frac{1}{\delta}\left[1-\frac{\Gamma(1-\delta)\Gamma(1+\nu)}{\Gamma(1-\delta+\nu)}x^\delta\right]. \quad (26)$$

Note also that $1/\tilde{\delta}$ coincides approximately with $1/\delta$ when $\delta \neq 0$ and $x \to 0$. However, as $\delta \to 0$, the value of $1/\tilde{\delta}$ is not singular:

$$\frac{1}{\tilde{\delta}} \to \ln\left(\frac{1}{x}\right) - [\Psi(1+\nu) - \Psi(1)]. \quad (27)$$

Thus, (24) together with the well-known expressions of the anomalous dimensions $\gamma_{ab}^{(0),n}$ and $\gamma_{ab}^{(1),n}$ $(a,b = q,g)$, and the coefficient functions $B_2^{a,n}$ and $R_{\rm L}^{a,n}$ $(a = q,g)$ (see [55–57], respectively, and references therein) gives a possibility to extract the SF $F_{\rm L}$ at small x values. The calculations are based on precise experimental data of the SF F_2 and its derivatives ${\rm d}F_2/{\rm d}\ln(Q^2)$ and $\delta \equiv {\rm d}\ln F_2/{\rm d}\ln(1/x)$.

For concrete δ values, (24) simplifies essentially (see [20]). For example, for $\delta = 0.3$ we obtain (for the number of active quarks $f = 4$ and the $\overline{\rm MS}$ scheme):

$$F_{\rm L}(x,Q^2) = \frac{0.84}{1+59.3a_{\rm s}(Q^2)} \times \left[\frac{{\rm d}F_2(0.48x,Q^2)}{{\rm d}\ln Q^2} + 3.59a_{\rm s}(Q^2)F_2(0.48x,Q^2)\right] + O(a_{\rm s}^2, x^{2-\delta}, a_{\rm s}x^{1-\delta}). \quad (28)$$

At arbitrary δ values, in real applications it is very useful to simplify (24) as follows. We keep the exact δ-dependence only for the leading order terms, which are very simple. In the NLO corrections we extract the terms $\sim 1/\tilde{\delta}$ which change strongly when $0 \le \delta \le 1$, and parameterize the remaining terms in the form: $a_i + b_i\delta + c_i\delta^2$. The coefficients a_i, b_i, c_i are fixed from the agreement of these parameterizations with the exact values of $\overline{\gamma}_{qa}^{(1),\eta}$ and $\overline{R}_{\rm L}^{a,\eta}$ at $\delta = 0$, 0.3 and 0.5. These exact values can be found in [20, 21, 33].

Then the approximate representation of (24) for arbitrary δ value has the form

$$F_{\rm L}(x,Q^2) = \frac{r(1+\delta)(\xi(\delta))^\delta}{\left(1+30a_{\rm s}(Q^2)\left[1/\tilde{\delta} - \frac{116}{45}\rho_1(\delta)\right]\right)} \times \left[\frac{{\rm d}F_2(x\,\xi(\delta),Q^2)}{{\rm d}\ln Q^2} + \frac{8}{3}\rho_2(\delta)a_{\rm s}(Q^2)F_2(x\,\xi(\delta),Q^2)\right] + O(a_{\rm s}^2, a_{\rm s}x, x^2), \quad (29)$$

where

$$r(\delta) = \frac{4\delta}{2+\delta+\delta^2}, \quad \xi(\delta) = \frac{r(\delta)}{r(1+\delta)}, \quad (30)$$
$$\rho_1(\delta) = 1+\delta+\delta^2/4, \quad \rho_2(\delta) = 1-2.39\delta+2.69\delta^2.$$

4 Comparison with the $F_{\rm L}$ experimental data

With the help of the results obtained in the previous section we have analyzed the experimental data for the SF $F_{\rm L}$[13] from the H1 [13, 18], NMC [58], CCFR [59, 60], BCDMS [61] Collaborations[14]. Note that we do not correct the CCFR data [59, 60] which have been obtained in the νN processes because the terms $\sim x \cdot m_c^2/Q^2$, which are different in the μN and the νN processes, are not so strong at low x.

We calculate the SF $F_{\rm L}$ as a sum of two types of contributions: the charm quark one $F_{\rm L}^c$ and the light quark one $F_{\rm L}^l$:

$$F_{\rm L} = F_{\rm L}^l + F_{\rm L}^c. \quad (31)$$

We use the expression (3) for the calculation of both the SF $F_{\rm L}^l$ and $F_{\rm L}^c$ in the following form (here $a_c = m_c^2/Q^2$):

$F_{\rm L}^l(x,Q^2)$

[12] In our formula (24) we are mostly interested in gluons, so we can apply $\nu = \nu_g \approx 4$ below

[13] Sometimes there are experimental data for the ratio $R = \sigma_{\rm L}/\sigma_{\rm T}$ which can be recalculated for the SF $F_{\rm L}$ because $F_{\rm L} = F_2R/(1+R)$

[14] We do not use the experimental data of R from the SLAC [62, 63], EM [64] and CDHSW [65] Collaborations because they are obtained at quite large x values

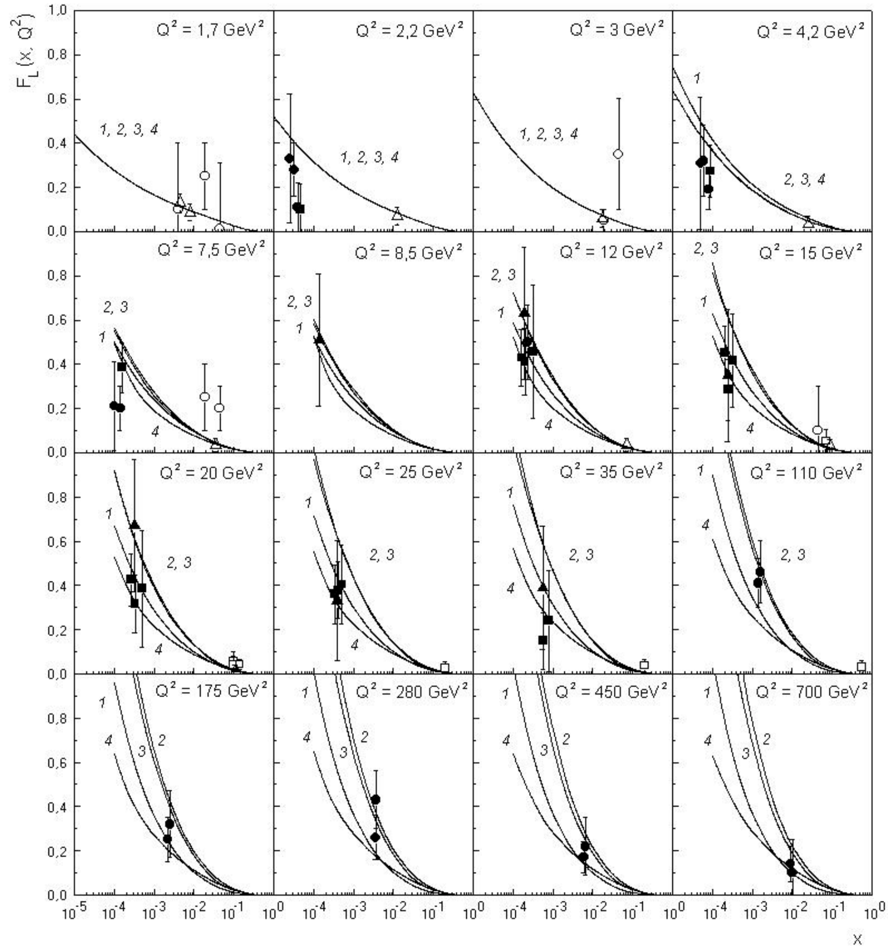

Fig. 2. The structure function $F_{\mathrm{L}}(x, Q^2)$ as a function of x for different values of Q^2 compared to the experimental data. The H1 data are as follows: the first 1997 ones [13], new 2001 ones [15] and preliminary ones [18] are shown as black triangles, circles and squares, respectively. The data of the NM [58], CCFR [60] and BCDMS [61] Collaborations are shown as white triangles, circles and squares, respectively. Curves 1, 2, 3 and 4 correspond to the SF obtained in the perturbative QCD with the GRV [43] quark and gluon densities at the LO approximation and to the SF obtained in the k_{T}-factorization approach with the JB (at $Q_0^2 = 4\,\mathrm{GeV}^2$) [68], KMS [72] and GBW [69] parameterizations for the unintegrated gluon distribution

$$
\begin{aligned}
&= \sum_{f=uds} e_f^2 \left[\int_x^1 \frac{\mathrm{d}z}{z} \hat{C}^g_{\mathrm{L}}\left(\frac{x}{z}, Q^2, 0\right) z f_g(z, Q_0^2) \right.\\
&+ \sum_{i=1}^{2} \int_{z^{(i)}_{\min}}^{z^{(i)}_{\max}} \frac{\mathrm{d}z}{z} \int_{k^{2(i)}_{\perp\min}}^{k^{2(i)}_{\perp\max}} \mathrm{d}k_\perp^2 \hat{C}^g_{\mathrm{L}}\left(\frac{x}{z}, Q^2, k_\perp^2\right)\\
&\left. \times\, \Phi(z, k_\perp^2, Q_0^2) \right], \qquad (32)
\end{aligned}
$$

$$
\begin{aligned}
&F^c_{\mathrm{L}}(x, Q^2)\\
&= e_c^2 \left[\int_{x(1+4a_c)}^1 \frac{\mathrm{d}z}{z} \hat{C}^g_{\mathrm{L}}\left(\frac{x}{z}, m_c^2, Q^2, 0\right) z f_g(z, Q_0^2) \right.\\
&+ \sum_{i=1}^{2} \int_{z^{(i)}_{\min}}^{z^{(i)}_{\max}} \frac{\mathrm{d}z}{z} \int_{k^{2(i)}_{\perp\min}}^{k^{2(i)}_{\perp\max}} \mathrm{d}k_\perp^2 \hat{C}^g_{\mathrm{L}}\left(\frac{x}{z}, m_c^2, Q^2, k_\perp^2\right)\\
&\left. \times\, \Phi(z, k_\perp^2, Q_0^2) \right], \qquad (33)
\end{aligned}
$$

where $\hat{C}^g_{\mathrm{L}}(x_B, Q^2, m_c^2, k_\perp^2)$ are given by (9) and (14).

The integration limits in expression (33) have the following values:

$$
\begin{aligned}
&z^{(1)}_{\min} = x\left(1 + 4a_c + \frac{Q_0^2}{Q^2}\right),\ z^{(1)}_{\max} = 2x(1 + 2a_c);\\
&k^{2(1)}_{\perp\min} = Q_0^2, \quad k^{2(1)}_{\perp\max} = \left(\frac{z}{x} - (1 + 4a_c)\right) Q^2;\\
&z^{(2)}_{\min} = 2x(1 + 2a_c), \quad z^{(2)}_{\max} = 1;\\
&k^{2(2)}_{\perp\min} = Q_0^2, \quad k^{2(2)}_{\perp\max} = Q^2. \qquad (34)
\end{aligned}
$$

The ranges of integration correspond to positive values of the square roots in expressions (10), (11), (15) and (16) and should also obey the kinematic restriction ($z \le (1 + 4a_c + b)^{-1}$) following from the condition $\beta^2 \ge 0$ (see (10)–(12)). In (32) the ranges (34) are used at $a_c = 0$.

In Fig. 2 we show the SF F_{L} as a function of x for different values of Q^2 in comparison with the H1 experimental data sets: the old one of [13] (black triangles), the last year's one of [15] (black squares) and the new preliminary one of [18] (black circles), and also with NMC [58] (white triangles), CCFR [59] (white circles) and BCDMS [61] data (white squares). For comparison with these data we present the results of the calculation with three different parameterizations for the unintegrated gluon distribution $\Phi(x, k_\perp^2, Q_0^2)$ at $Q_0^2 = 4\,\mathrm{GeV}^2$. All of them, the Kwiecinski–Martin–Stasto (KMS) one [72], the Blumlein (JB) one [68], and the Golec-Biernat and Wusthoff (GBW) one [72], have already been used in our previous work [26] and reviewed there.

There are several other popular parameterizations (see, for example, those of Kimber–Martin–Ryskin (KMR) [70] and Jung–Salam (JS) [71]), which are not used in our study mostly because of technical difficulties. Note that all the above parameterizations give quite similar results

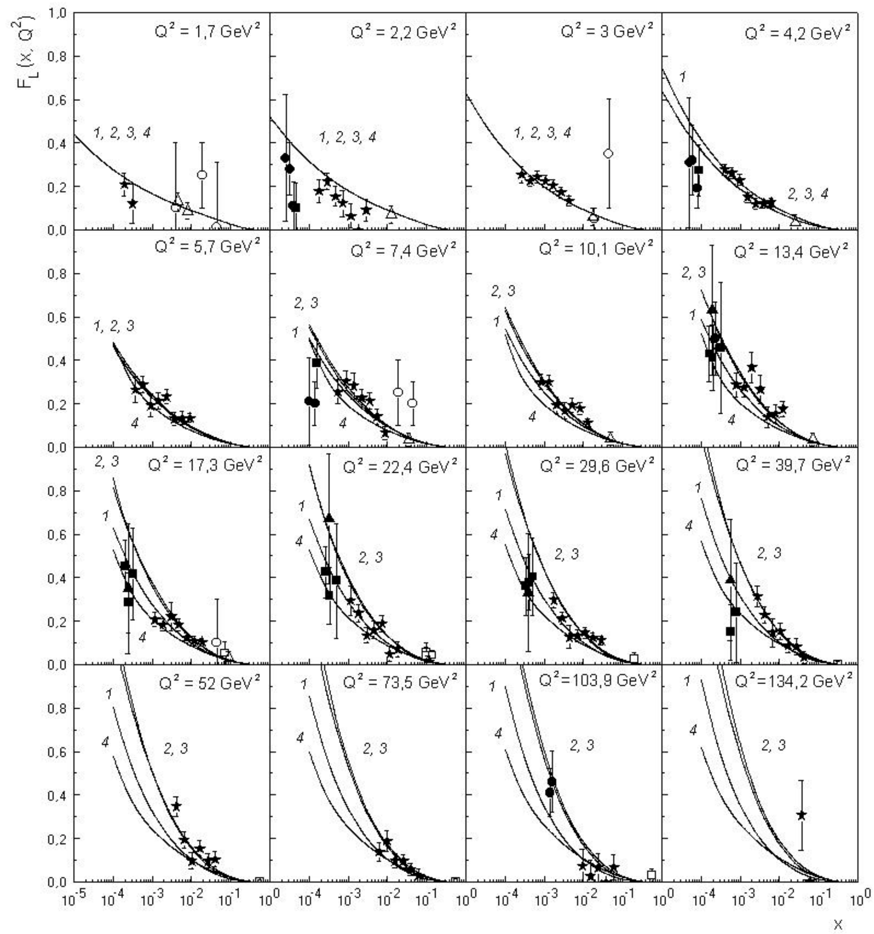

Fig. 3. The structure function $F_{\mathrm{L}}(x, Q^2)$ as a function of x for different values of Q^2. To compare with Fig. 2 "experimental data" (see [19–21] and Sect. 3) are added as black stars. The "experimental data" values depend mostly on the derivative $\mathrm{d}F_2(x, Q^2)/\mathrm{d}\ln Q^2$, which data are known at a little bit different Q^2 values (see [15]). So, the "experimental data" obtained at 12, 15, 20, 25 and 35 GeV^2 are presented here at 13.4, 15.3, 22.4, 29.6 and 39.7 GeV^2, respectively

except, perhaps, for the contributions from the small $k_\perp^2$ range: $k_\perp^2 \leq 1\,\mathrm{GeV}^2$ (see [11] and references therein). As we use $Q_0^2 = 4\,\mathrm{GeV}^2$ in the study of the SF F_{L}, our results depend very slightly on the small $k_\perp^2$ range of the parameterizations. In the case of the JB, GBW and KMS sets this observation is supported below by our results and we expect that the application of the KMR and JS sets should not strongly change our results.

The differences observed between the curves 2, 3 and 4 are due to the different behavior of the unintegrated gluon distribution as a function of x and $k_\perp$. We can see that the SF F_{L} results obtained in the k_{T}-factorization approach with the KMS and JB parameterizations are close to each other[15] and higher than the ones obtained in the pure perturbative QCD with the GRV quark and gluon densities in the leading order (LO) approximation. Otherwise, the F_{L} results based on the k_{T}-factorization approach with the GBW parameterization are quite close to the pure QCD predictions[16]: such is, indeed, the case because the GBW model has deviations from the gluon part of perturbative QCD only at quite low Q^2 values. We note that a greater part of the difference between the predictions of the pure QCD and the GRW model comes from contributions of the quark densities, which are absent in the GRW model. These quark contributions are responsible for the intermediate place of the curves corresponding to pure QCD predictions: they lie between the curves corresponding to the GRW model and the KMS and/or JB ones.

Thus, the predictions of perturbative QCD and the ones based on the k_{T}-factorization approach are in agreement with each other and with all data within the modern experimental uncertainties. So, the possible high values of high-twist corrections to the SF F_{L} predicted in [40] can be important only at low Q^2 values: $Q^2 \leq Q_0^2 = 4\,\mathrm{GeV}^2$.

Figure 3 is similar to Fig. 2 with one exception: we add the "experimental data" obtained using the relation between the SF $F_{\mathrm{L}}(x, Q^2)$, $F_2(x, Q^2)$ and $\mathrm{d}F_2(x, Q^2)/\mathrm{d}\ln Q^2$ (see Sect, 3) as black stars. Since the corresponding data for the SF $F_2(x, Q^2)$ and $\mathrm{d}F_2(x, Q^2)/\mathrm{d}\ln Q^2$ are essentially more precise (see [15]) to compare with the preliminary data [18] for F_{L}, the "experimental data" have strongly suppressed uncertainties. As is shown in Fig. 3, there is a very good agreement between the new preliminary data [18], the "experimental data" and predictions of perturbative QCD and the k_{T}-factorization approach.

To estimate the value of the charm mass effect, we recalculate the SF F_{L}^c also in the massless approximation similar to (32). In Fig. 4, we show the importance of the

[15] Note that very similar results have also been obtained for Ryskin–Shabelsky parameterization [72] (see also [73])

[16] This fact is also evident from quite the large value of $Q_0^2 = 4\,\mathrm{GeV}^2$ chosen here

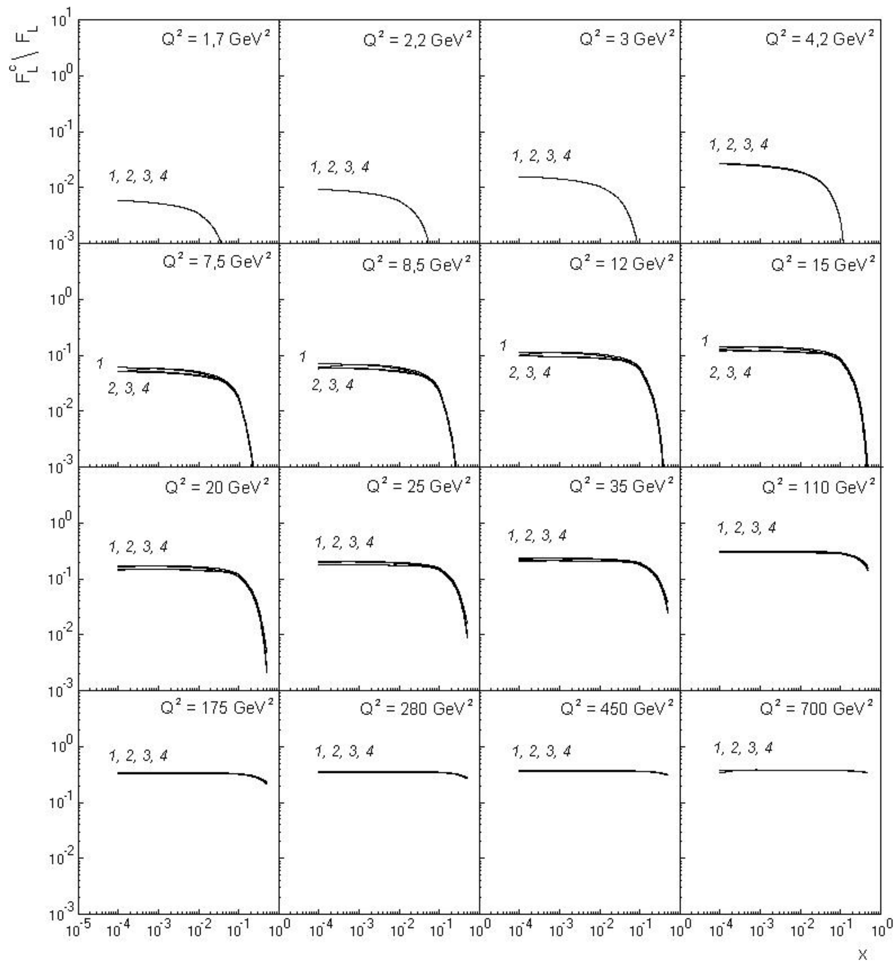

Fig. 4. The ratio $F_{\mathrm{L}}^c(x,Q^2)/F_{\mathrm{L}}(x,Q^2)$ as a function of x for different values of Q^2. Curves 1, 2, 3 and 4 correspond to the ratio obtained in the perturbative QCD with the GRV [43] quark and gluon densities at the LO approximation and to the SF obtained in the k_{T}-factorization approach with the JB (at $Q_0^2 = 4\,\mathrm{GeV}^2$) [68], KMS [72] and GBW [69] parameterizations for the unintegrated gluon distribution

exact m_c-dependence in the hard SF of F_{L}^c to be compared with its massless approximation where we should have $F_{\mathrm{L}}^c(m_c = 0)/F_{\mathrm{L}}(m_c = 0) = 2/5$. As one can see in Fig. 4, the ratio $F_{\mathrm{L}}^c/F_{\mathrm{L}}$ goes to the massless limit 2/5 only at asymptotically large Q^2 values.

5 Conclusions

In the framework of the k_{T}-factorization approach we have applied the results of the calculation of the perturbative parts for the structure functions F_{L} and F_{L}^c for a gluon target having a non-zero momentum squared, in the process of photon–gluon fusion, to the analysis of the present data for the structure function F_{L}[17]. The analysis has been performed with several parameterizations of the unintegrated gluon distributions, for comparison. We have found a good agreement between all existing experimental data, the predictions for F_{L} obtained from the relation between the SF $F_{\mathrm{L}}(x,Q^2)$, $F_2(x,Q^2)$ and $\mathrm{d}F_2(x,Q^2)/\mathrm{d}\ln Q^2$, and the results obtained in the framework of perturbative QCD and the ones based on the k_{T}-factorization approach with the three different parameterizations of the unintegrated gluon distributions.

We note that we use the leading order of the k_{T}-factorization approach. As has been noted in [11], the k_{T}-factorization includes (at least some of the) NLO corrections of the collinear approach. So, the good agreement observed above between k_{T}-factorization contributions and the LO and NLO predictions based on the relation between the SFs $F_{\mathrm{L}}(x,Q^2)$, $F_2(x,Q^2)$ and $\mathrm{d}F_2(x,Q^2)/\mathrm{d}\ln Q^2$ is no surprise.

As we know, there are already investigations to extend the k_{T}-factorization approach beyond the LO approximation. However, the subject of the investigations is essentially the one above in our formalism, and a review of these studies can be found in [11].

We note that it could also be very useful to evaluate the SF F_2 itself[18] and the derivatives of F_2 with respect to the logarithms of $1/x$ and Q^2 with our expressions using the unintegrated gluons. We are considering to present this work and also the predictions for the ratio $R = \sigma_{\mathrm{L}}/\sigma_{\mathrm{T}}$ in a forthcoming article.

The consideration of the SF F_2 in the framework of the leading-twist approximation of perturbative QCD (i.e. for "pure" perturbative QCD) leads to a very good agreement (see [45] and references therein) with the HERA data at

[17] In [26], we have also obtained quite a large contribution of the SF F_{L}^c at low x and high Q^2 ($Q^2 \geq 30\,\mathrm{GeV}^2$)

[18] A study of the SF F_2 in the framework of the k_{T}-factorization has already been carried out in [74]

low x and $Q^2 \geq 1.5\,\mathrm{GeV}^2$. The agreement improves at lower Q^2 when higher-twist terms are taken into account [38, 39]. It has been studied in [38, 45] that the SF F_2 at low Q^2 is sensitive to the small-x behavior of the quark distributions. Thus, our future analysis of F_2 in a broad Q^2 range in the framework of the k_T-factorization approach should require the incorporation of the parameterizations of the unintegrated quark densities introduced recently (see [11, 70] and references therein).

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